Petri Nets for Modelling Norms of Social Exchange

Georg P. Mueller

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1. Introduction

In recent years, network analysis has become a standard methodology for exploring and describing group relations and social exchange (Borgatti et al. 2013; Scott 1998). As exemplified by Fig. 1, it is based on nodes and arcs, representing actors and their mutual relations. In spite of its merits and great scientific success, social network analysis has some serious drawbacks if it is used for analysing social exchange and the related norms of behaviour. First, it mainly refers to just *one* type of relations: power, love, or influence, to give a few examples. The reality of social exchange is obviously more complicated: trade relations between two companies imply e.g. not only a flow of a commodities from A to B but also a financial counterflow from B to A. Second, the different relations between the connected nodes are not *independent,* as often suggested in classical network analysis: if in the previous example the financial flow from B to A is interrupted, the supply of commodities from A to B is also stopped. Third, classical network analysis is rather *static* and lacks the conceptual tools to represent processes, i.e. sequences of events. For example, in the case of delivery on credit, the commodity flow from A to B starts first and the financial counterflow from B to A is delayed. Conversely, in the case of advance payment, the sequence of flows is just the reverse.

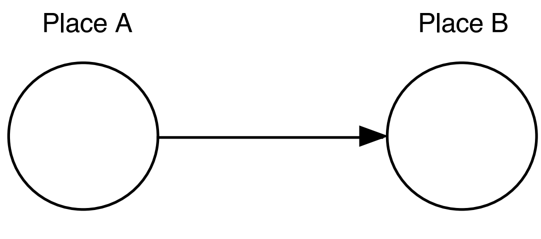


Fig. 1: An elementary dyad in traditional network analysis

In order to tackle the deficiencies of the traditional social network analysis, the author proposes to use the *Petri net* approach. It was invented by Carl Adam Petri in the 1960s for studying parallel processes in computer systems. However, its basic concepts like *tokens, places,* and *transitions* are general enough to be also used for modelling administrative and industrial processes (Van der Aalst, Stahl 2011; Heitsch et al. 2001). Nonetheless, applications in *sociological* research are still rare: among other scholars (e.g. Köhler, Rölke 2001) the author used it in an earlier publication (Mueller 2014) for studying the enactment of anti-smoking laws. In the present paper he first presents the main concepts of sociological Petri net analysis and subsequently uses them for studying a classic topic of social exchange theory: the norms of the Melanesian Kula trade, as described by Bronislaw Malinowski (2010: chap. 3).

2. Petri nets: A short overview

2.1 The descriptive tools of Petri nets

There are many different versions of Petri net analyses. Nonetheless, they have three basic elements in common (Reisig 1985: chap. 1; Desel, Reisig 1998):

a) *Tokens,* which may be anything that can be moved from one place, person, or organisation to another: cheques, commercial goods, legislative proposals, etc. In Petri net diagrams like Fig. 2 they are generally represented as *black dots.*
b) *Places,* which can host tokens. They are indicated by big circles (see Fig. 2) and define the location of tokens. The bearing capacity of places is generally limited: *small empty circles* indicate available places.
c) *Transitions,* indicated by rectangular bars (see Fig. 2). At regular time intervals they transfer ("fire") a token from an original to a connected target place - provided there is free space at the target and a token at the origin.

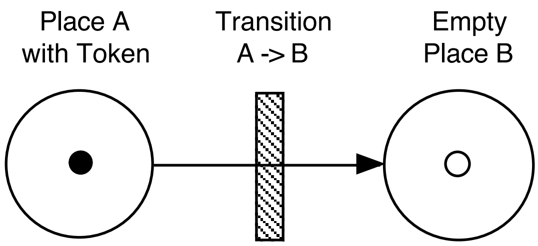


Fig. 2: An exemplary Petri net with places, tokens, and transitions

Depending on the phenomena to be modelled, the three basic elements of the Petri net approach can be supplemented and modified. Tokens may e.g. assume a *time-dependent value v,* like in Fig. 3, which allows the researcher to construct *timed Petri nets* (Wang 1998). v may be e.g. the changing commercial value of a good, which is only sold and exchanged with others, if it has reached a certain value ε (see Fig. 3). This way, the transition from A to B becomes a conditional event, which is accordingly specified in the Petri net diagram. Often v is the clock time, which triggers or inhibits other processes.

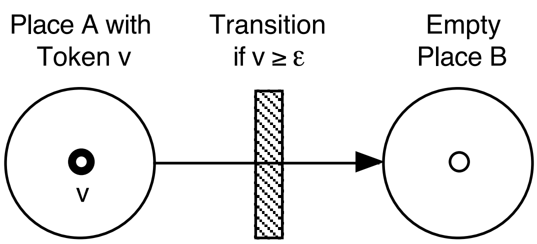


Fig. 3: A timed Petri net with transitions of time-dependent tokens

Tokens which are not time-dependent may too influence parallel processes by triggering and inhibiting the related transitions. Fig. 4 presents two equivalent mechanisms with the related symbols: if there is a token at place D, the transition A -> B is *inhibited.* If there is *no* token at D, the transition from A to B is possible, provided there are tokens at A and spaces at B. An inverse logic applies to place C: if it holds *no* token, the transition A -> B is inhibited. If there is a token at C, the transition from A to B is *enabled,* provided there are tokens at A and spaces at B. Thus, *before* the transition D -> C, the transition A -> B is locked by two redundant mechanisms. *After* the transition D -> C, the transition A -> B is enabled by the same two mechanisms. They are obviously useful for modelling social exchange, where there are generally two flows of items which are reverse and interdependent: in Fig. 4, the transfer A -> B is only possible after the realisation of the transition D -> C. This obviously also reflects an asymmetry of power between the actors controlling the different flows: the actor in control of the transition A -> B expects his/her counterpart to first move a token from D to C before he/she triggers the return flow from A to B.

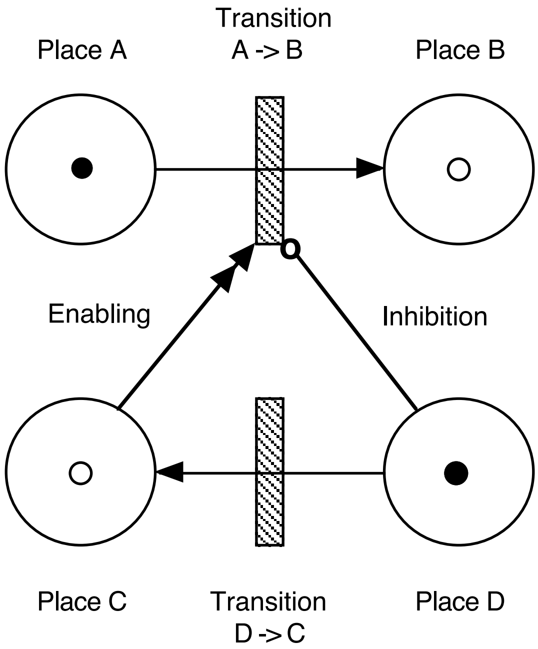


Fig. 4: A Petri net with enabling and inhibition of transitions

2.2 The usage of Petri nets for the analysis of social exchange

If applied to social exchange, Petri net models may allow to tackle to following scientific questions and goals:

a) *Precise descriptions* of existing institutions of social exchange and their functional design.
b) *Stress tests* for revealing the conditions under which institutions of social exchange fail to function in the way they have been designed. The number of norm violations or institutional breakdowns are typical indicators, which may vary by the workload or the number of participating actors.
c) *Comparisons of institutions* with regard to their performance under different conditions. The performance may e.g. be measured by the number of transactions per unit of time, which an institution is able to perform.

d) *Design of new institutions* of exchange. This kind of social engineering includes precise descriptions, stress tests, optimisation of performance, and comparisons with similar existing institutions.

Obviously, the previous analyses are more theoretical than empirical, since they are focussed on institutions and conditions which are often only virtual and do not (yet) exist in the real world. Thus the author suggests to use *computer simulation* in order to explore and compare Petri net models. It is flexible with regard to the changes in the analysed scenarios and easy to use for sociologists, who are generally less trained in mathematics. More demanding alternatives are calculations with *matrix algebra* or *graphical analyses* of sequential net states (Wang 1998: chap. 2.5).

3. A Petri net analysis of the Kula ring

3.1 Introductory remarks

Nearly one hundred years ago, the ethnographer Bronislaw Malinowski (2010 [1922]) published his famous book *Argonauts of the Western Pacific*. In chapter 3 of this work, he describes the Kula trade in Melanesia, which has inspired many other social scientists like Blau (1967, p. 93), Mauss (1990, p. 53 ff.), Leach and Leach (1983), and more recently Ziegler (1990, 2007, 2008). Kula is a symbolic exchange of prestigious goods, where decorated necklaces circulate clockwise and beautiful arm shells in the opposite direction between islands east of Papua New Guinea. Kula accompanies conventional trade with agricultural goods and manufactured products and serves for creating social cohesion and trust (Ziegler 2008: 209). Typically for Kula, there is always a temporal delay between the receipt of a gift and the presentation of an equivalent return gift. The first transaction intentionally creates a debt, which *later* has to be compensated by a reverse transaction (Gregory 1983, p. 115). Thus, Malinowski observed a system of *delayed* reciprocity, which is different from *immediate* reciprocity, where gifts and return gifts are exchanged at the same occasion.

In the following sections the author first attempts to represent the delayed reciprocity of the Kula trade as a Petri net in order to compare it with the alternative of immediate reciprocity, which too is modelled as a Petri net. Subsequently he performs stress tests for both models in order to explore their performance if Kula goods are running short. Such shortages may occur if merchants are extending their network (Ziegler 2008: 109) or illicitly hoarding Kula goods (Ziegler 2008: 109), or if new trade partners with no Kula goods enter into the system. The focus of these stress tests is first on the *maintenance of reciprocity* designed for improving social cohesion, second on the *integration of newcomers,* and third on possible *breakdowns* of the whole exchange system. The respective theoretical investigations are performed with computer simulations: an Excel program runs the Kula trade with the respective norms of reciprocity over a period of 200 units of time and analyses the results for different initial distributions of Kula goods.

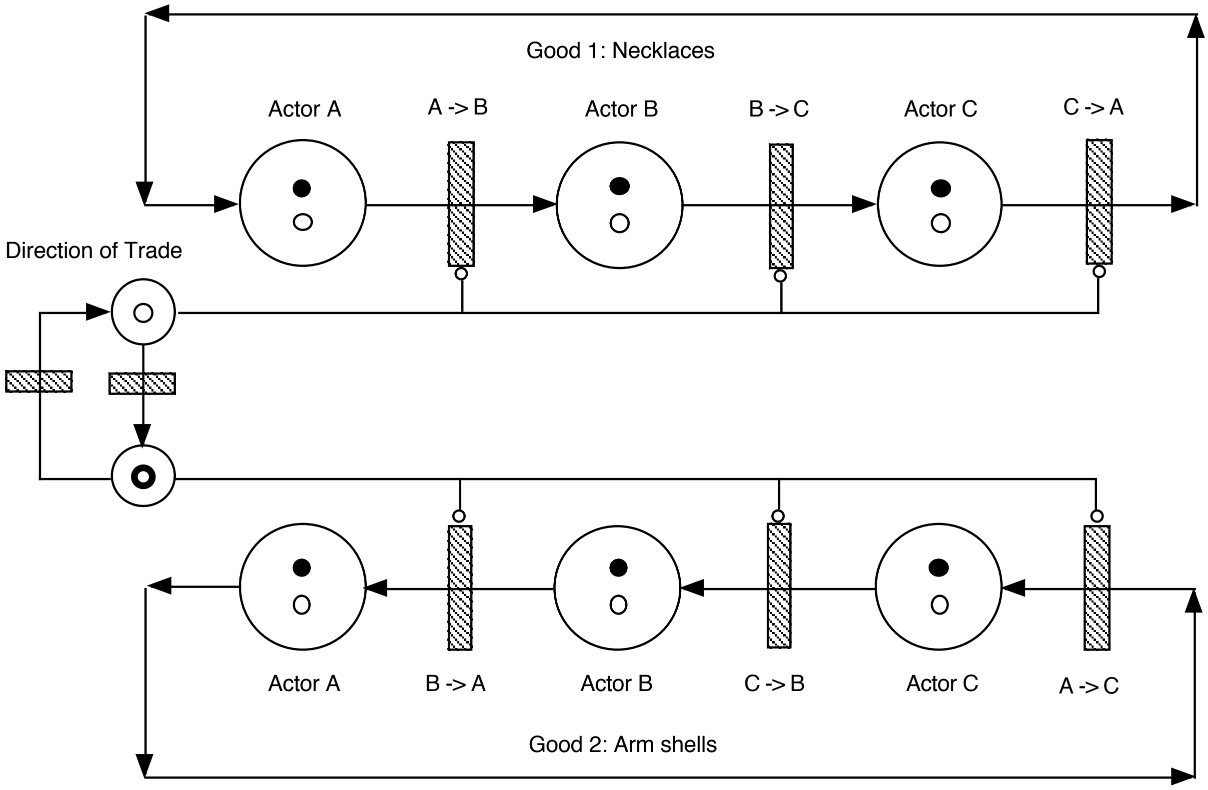


Fig. 5a: Delayed reciprocity as the original Kula model

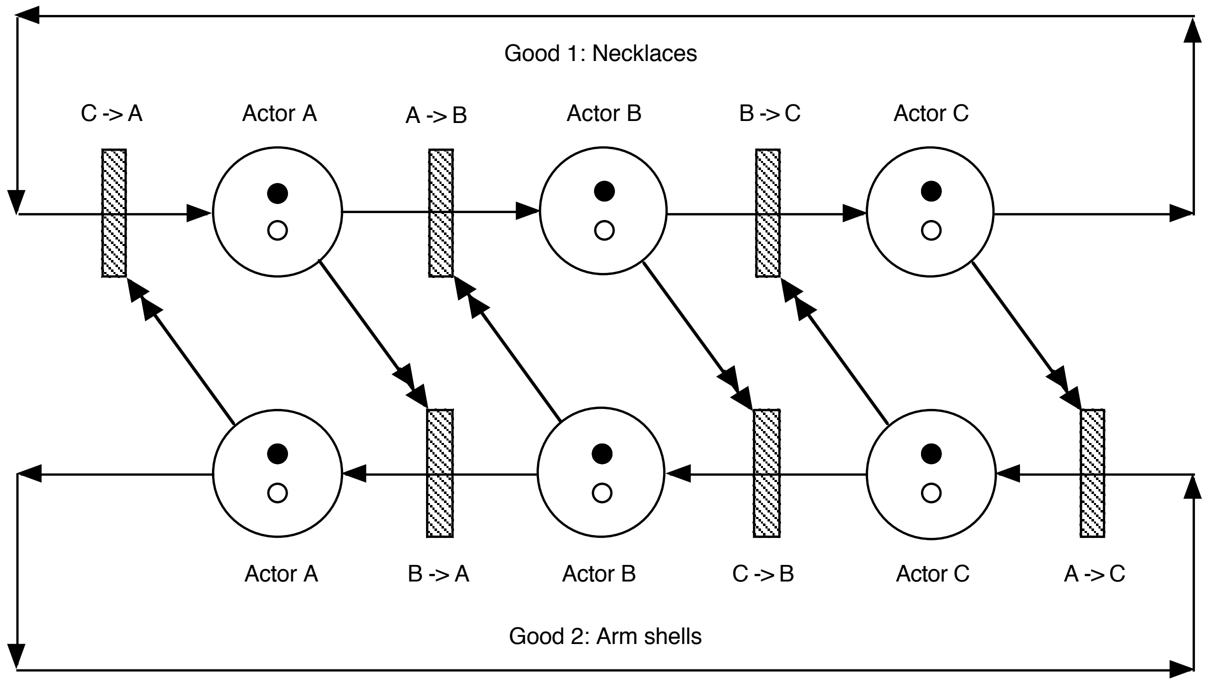


Fig. 5b: Immediate reciprocity as an alternative model

3.2 Two models of the Kula trade

Figure 5a presents the original Kula trade – for reasons of simplification with only three trade partners A, B, and C. Each of the three actors is able to present a necklace (good 1) to his or her neighbour. As conceptualised by the principle of *delayed* reciprocity, the return gifts in terms of arm shells are blocked by a time dependent token called "Direction of Trade". One time step later, this token is in the alternate position, where it inhibits the trade with good 1 and releases the exchange of good 2 (arm shells). Each of the actors A, B, and C is now able to fulfil his/her obligation of reciprocity, which however may be impossible in the case of a shortage of good 2. Thus the delayed reciprocity does not prevent violations of the norms of exchange.

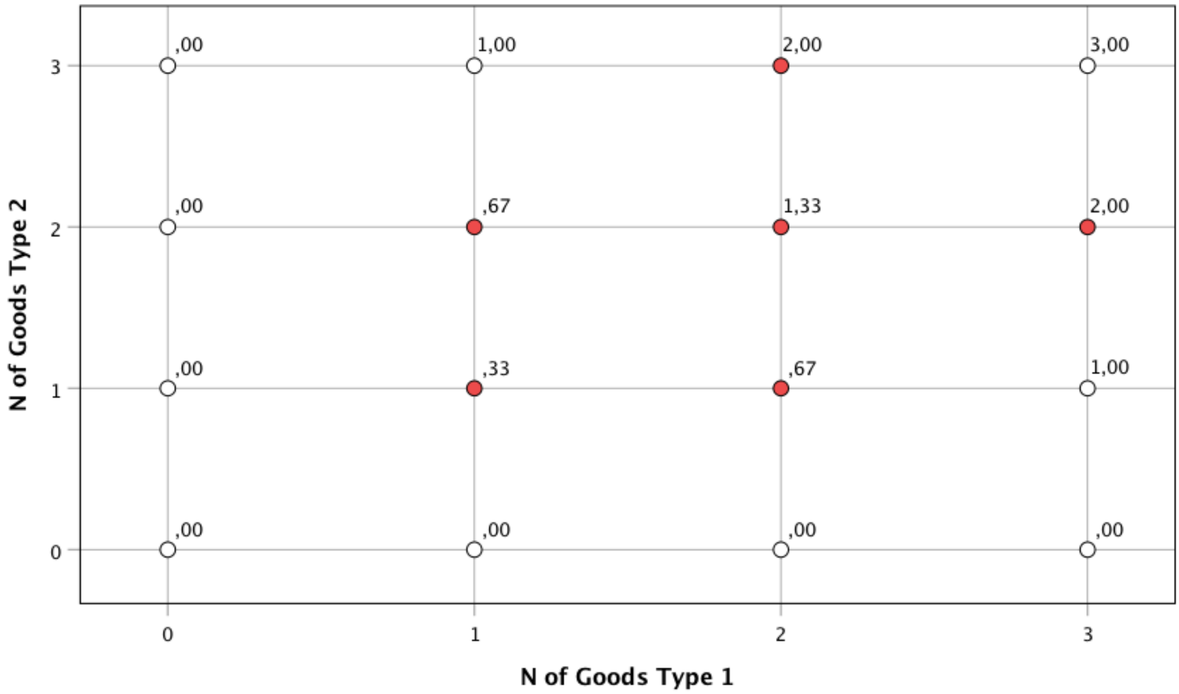
Figure 5b shows the counterfactual alternative of a Kula trade with *immediate* reciprocity: whenever two actors meet, they must exchange gifts before doing barter with commercial goods. If by a shortage of arm shells or necklaces one of the traders cannot offer a gift, the other will abstain from doing so: if actor A has e.g. no necklaces, the transfer of arm shells from B to A is not enabled, even if actor B is in possession of the respective good. Similarly, if actor B has no arm shells to give to A, the transfer of necklaces from A to B is disabled, independently of the stocks of actor A. Thus, according to Fig. 5b, there is either a two-way exchange of gifts or no exchange at all. In the first case the obligation of immediate reciprocity is fulfilled, whereas in the second the norm is considered as being violated.

3.3 Comparisons of the two models of reciprocity

Exchanging gifts is a symbolic act of mutual liking and consequently fosters *social cohesion,* which is further enhanced by respecting the related social norms of giving and receiving presents. Thus, in this section we are using the mean number of exchange cycles between the actors A, B, and C (see Figs. 5a, b), which are conforming to the specific norms of reciprocity. An exchange cycle fulfilling the norm of *immediate* reciprocity comprises just *one* encounter, where a gift and an immediate return gift are simultaneously exchanged. An exchange cycle fulfilling the norm of *delayed* reciprocity requires *two* successive encounters, where first a gift and one unit of time later a return gift are exchanged by the trading partners.

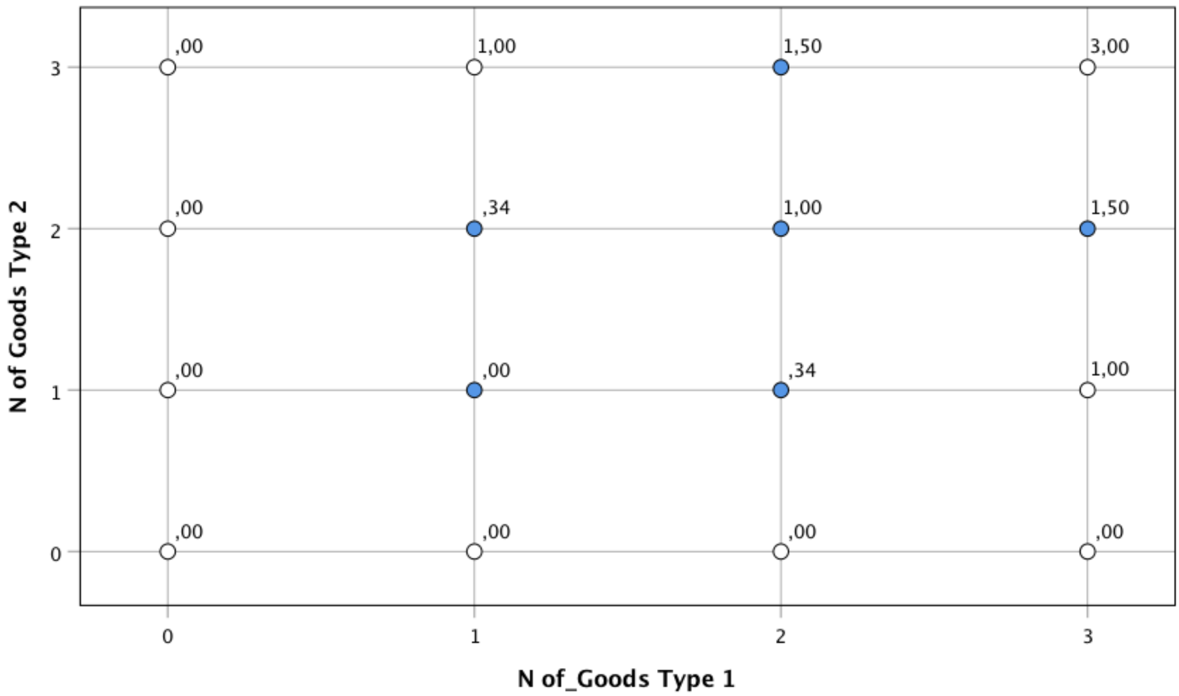
As long as there are 3 tokens of good 1 and 3 tokens of good 2 in the respective Petri nets of Figs. 5a and 5b, the maximum of both types of reciprocities is reached. If there are less tokens available, the reciprocities diminish and become dependent on the initial distribution of tokens between the three actors and the length of the computer simulations. For this reason the cohesion scores of Figs. 6a and 6b are equal to the *mean* number of reciprocities, equally averaged over all possible initial distributions of tokens and 200 simulated units of time.

A comparison between Figs. 6a and 6b reveals no difference between immediate and delayed reciprocity if one of the two goods is *completely* lacking: not very surprisingly social cohesion is for both types of reciprocity equal to *zero.* A similar observation holds for full endowment with goods: if 3 items of good 1 and 3 items of good 2 are available, both types of reciprocity norms lead to the maximum cohesion score 3. The farer away the endowment with goods from this optimal point, the lower the cohesion resulting from both types of reciprocities. However, the comparison between Figs. 6a and 6b shows that the cohesion from *delayed* reciprocity is always *equal* to or *better* than the corresponding cohesion from *immediate* reciprocity.



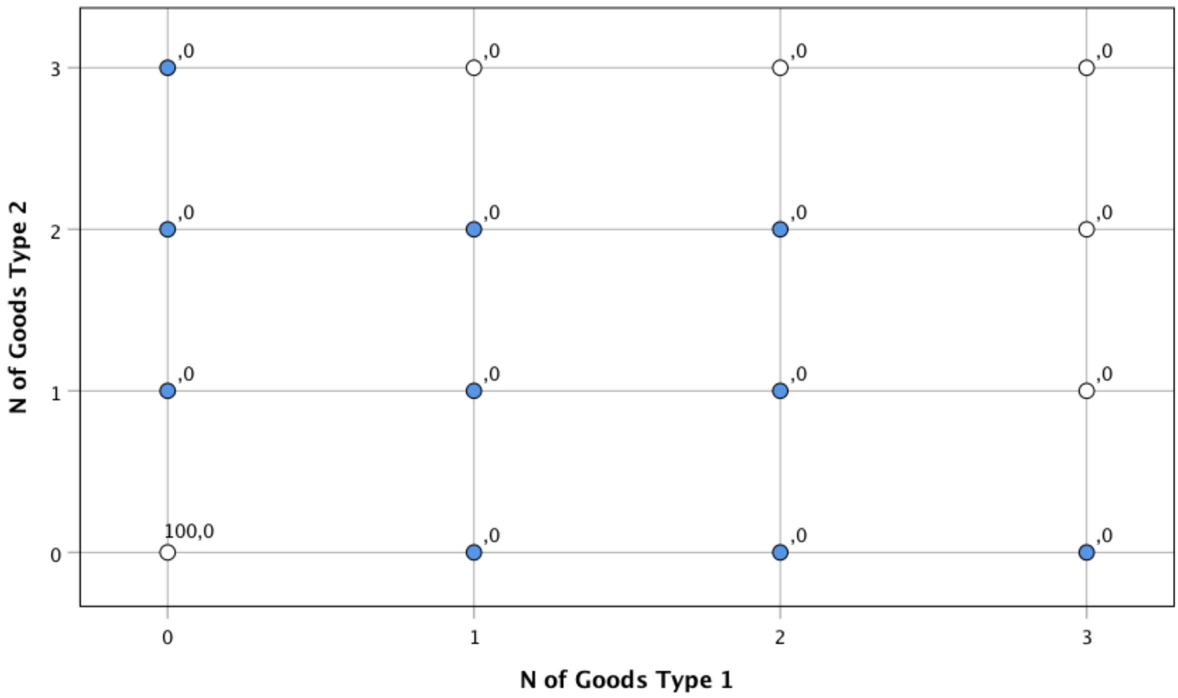
<u>Legend</u>: Red: higher than reference (= Fig. 6b); white: identical with reference (= Fig. 6b).
<u>Operationalisation</u>: Social cohesion = Mean number of exchange cycles between actors A, B, and C which correspond to the norm of *delayed* reciprocity. Minimum = 0, maximum = 3.

Fig. 6a: Social cohesion for delayed reciprocity



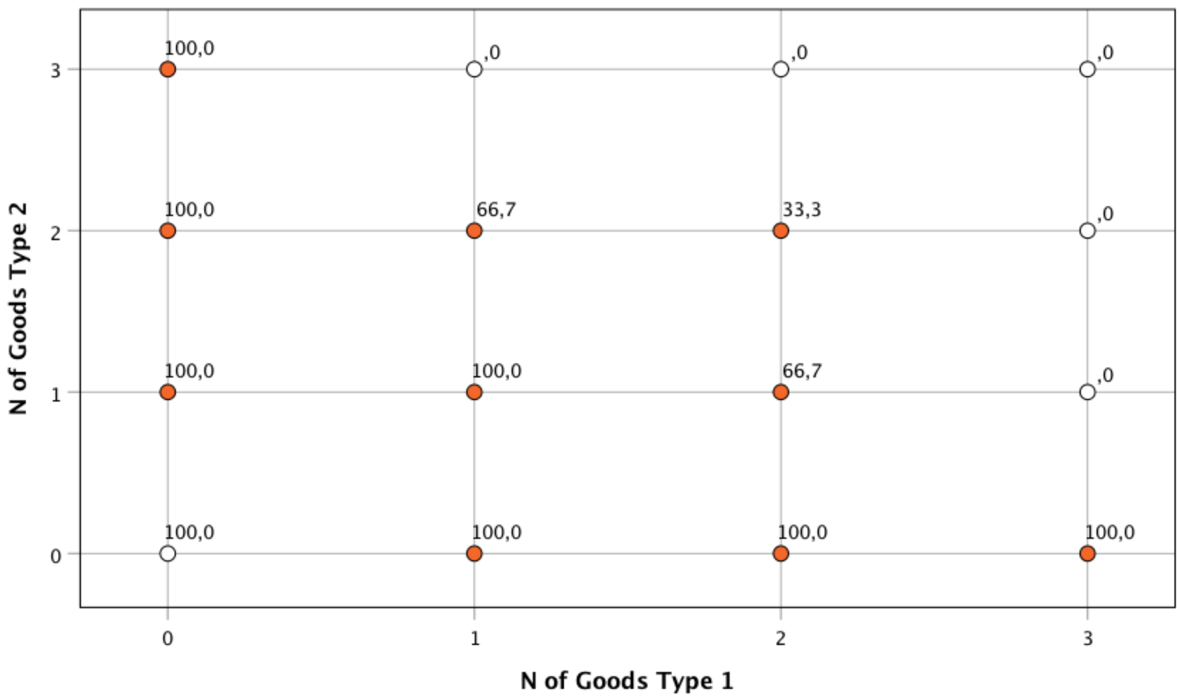
<u>Legend</u>: Blue: lower than reference (= Fig. 6a); white: identical with reference (= Fig. 6a).
<u>Operationalisation</u>: Social cohesion = Mean number of exchange cycles between actors A, B, and C which correspond to the norm of *immediate* reciprocity. Minimum = 0, maximum = 3.

Fig. 6b: Social cohesion for immediate reciprocity



<u>Reference</u>: Percentage of breakdowns for immediate reciprocity: see Fig. 7b. <u>Legend</u>: Blue: lower than reference; white: identical with reference.

Fig. 7a: Delayed reciprocity: Percentage of initial distributions leading to breakdown



<u>Reference</u>: Percentage of breakdowns for delayed reciprocity: see Fig. 7a. <u>Legend</u>: Red: higher than reference; white: identical with reference.

Fig. 7b: Immediate reciprocity: Percentage of initial distributions leading to breakdown

A part of the low cohesion scores is due to the *breakdown* of the institution of reciprocal exchange of Kula gifts, which means that giving and receiving goods of type 1 and type 2 stops shortly after the start of the simulation. Its occurrence depends again on the initial distribution of the available goods among the actors A, B, and C. Consequently, we are analysing in Figs. 7a and 7b the percentage of initial distributions which result in this kind of deadlock. It obviously varies between 0% and 100%. As Fig. 7a shows, breakdowns hardly ever occur for *delayed* reciprocity. The only trivial exception is the *total absence* of any Kula good. As soon as there exists at least *one* token of one of the goods, it is transferred from one actor to the next, even if there is no reciprocity due to a total lack of the other good. Consequently, the complete breakdown of the exchange system can be avoided. This situation is completely different from *immediate* reciprocity: if one type of good is totally missing, the available other good can also *not* be traded because of the lack of an *immediate* return gift. As Fig. 7b shows, this situation results in 100% of cases in a breakdown. *Partial* deadlocks with frequencies lower than 100% still happen for milder forms of shortages of goods. Although the institution of immediate reciprocity can also be fail-safe – e.g. if one type of good is fully endowed with 3 tokens and the other with at least 1 token – it is clearly less robust than delayed reciprocity. For all those cases where according to Fig. 7b breakdowns reach the level of 100%, the *long-term survival* of the institution of immediate reciprocity is likely to be endangered.

The previously discussed breakdown of exchange is just one of the risks which threatens the long-term survival of an institution like Kula. Another risk is the failure to *recruit* and *integrate* new members in order to replace others who abandoned the Kula trade for reasons of bad health or old age. It is accentuated by the fact that newcomers may at the beginning lack the resources for really participating in the system. In order to assess this risk for immediate and delayed reciprocity, we simulated the mean long-term participation in good 1 and good 2 for actors having different initial endowments with the mentioned Kula goods. The results are presented in Tab. 1.

Table 1: Mean long-term participation in goods 1 and 2 of a newcomer, by his/her initial participation and the type of reciprocity

Initial participation:		**Immediate reciprocity:** Long-term participation:		**Delayed reciprocity:** Long-term participation:	
Good 1:	Good 2:	Good 1:	Good 2:	Good 1:	Good 2:
1.00	1.00	1.00	1.00	1.00	1.00
1.00	0.00	0.50	0.50	**1.00**	**0.66**
0.00	1.00	0.50	0.50	**0.66**	**1.00**
0.00	0.00	0.00	0.00	**0.66**	**0.66**

Legend: Long-term participation: Mean possession of goods over 200 simulated units of time. Bold: Higher than the corresponding figure of the other type of reciprocity.
Note: Initial participation of the two *non*-newcomers: Good 1 = 1, Good 2 = 1.

Under the rule of *immediate* reciprocity, a newcomer lacking *both goods* has no chance at all to get into the Kula trade: having nothing to offer to his/her trade partners, the person cannot get any goods from them. This exclusion is obviously different from the situation of a poor novice under the rule of *delayed* reciprocity. Here, the same person gets tokens from his/her neighbours, who hope to receive *later* return gifts, although this may not always be the case. In the long run, however, originally poor newcomers are in possession of 0.66 tokens of *both* goods (see Tab. 1), which can be exchanged with others and thus guarantee their partial integration into the Kula trade. Under the rule of *immediate* reciprocity, partial integration is only possible if a newcomer has one token either of good 1 or of good 2. Nonetheless, the resulting long-term participation equals only 0.50 for each of the goods, which is according to Tab. 1 clearly inferior to the situation of newcomers to the Kula system with delayed reciprocity. In sum, immediate reciprocity has a lower capacity to integrate newcomers, except for the trivial situation, where they are fully endowed with Kula resources (see Tab. 1, 1st line).

4. Summary and conclusions

This article started with a critique of traditional network analysis. For describing and analysing social exchange it is *too static,* mainly focused on *one type* of relations, and often neglecting the *interdependence* of different network ties. As an alternative, the author suggests using the tools of Petri nets. In order to show their applicability for such purposes, he presents an analysis of the Melanesian Kula trade, which was originally described by Malinowski. Petri nets are indeed useful for describing the rules of *delayed reciprocity,* which is a central feature of Kula. They can also be used for a precise description of an alternative fictitious Kula rule, i.e. *immediate* reciprocity. The author subsequently performs stress tests for both types of reciprocity, assuming that some of the trade partners are lacking resources, which are essential for participating in the Kula. Computer simulations reveal that delayed reciprocity has strategic advantages as compared to the alternative of immediate reciprocity. It guarantees more *social cohesion,* has a lower *risk of breakdowns,* and allows to *integrate newcomers* with no Kula resources. The latter two properties are crucial for any institution and may explain the observed long-term survival of the Kula trade: in spite of global modernisation, it still existed in the 1970s, i.e. sixty years after the ethnographic observations of Malinowski (Leach 1983).

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